בסיעתא דשמיא

2017

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2017 AP Calculus Exam

1. If $f(x)=(2x^2+5)^7$, then f'(x)=

Calculations: $7(4x)(2x^2+5)^6$ $28x(2x^2+5)^6$	

2. $\int \frac{1}{3x+12}$

Calculations: $\int \frac{1}{3x+12}dx=\frac{1}{3}\int \frac{1}{x+4}dx$ $\frac{1}{3}\ln\lvert x+4\rvert+C$	**Caveat:** Factor out then integrate.

3. $\frac{5-x}{x^3+2}$

Calculations:	

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$$f(x) = \frac{5-x}{x^3+2}$$

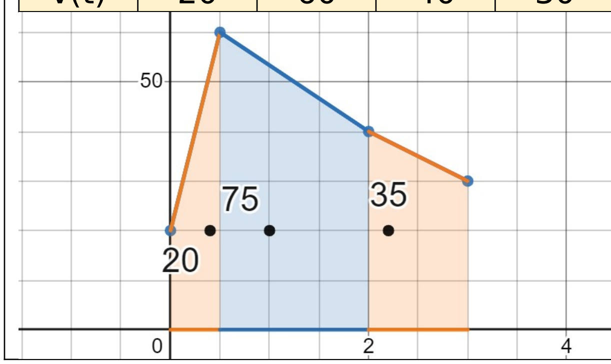
$$\frac{\left(x^3+2\right)(-1) - (5-x)\left(3x^2\right)}{\left(x^3+2\right)^2}$$

$$\frac{-x^3-2-15x^2+3x^3}{\left(x^3+2\right)^2} =$$

$$\frac{2x^3-15x^2-2}{\left(x^3+2\right)^2}$$

4. Trapezoidal Sum: The table gives the velocity v(t), in miles per hour, of a truck. Using a **Trapezoidal Sum** with n=3, what is the approximate distance, in miles, the truck traveled from t=0 to t=3.

t	0	.5	2	3
v(t)	20	60	40	30



Calculations:
(40)(.5)+(50)(1.5)+(35)(1)=130

$\frac{20+60}{2} \cdot \frac{1}{2}$ $\frac{80}{4}=20$	$\frac{60+40}{2} \cdot \frac{3}{2}$ $50 \cdot \frac{3}{2}=75$
$\frac{40+30}{2} \cdot 1$ ¿35	

5. If $f(x)=\sin\left(x^2+\pi\right)$, then $f'\left(\sqrt{2\pi}\right)$

Calculations:

$$f'(x) = 2x\cos\left(x^2+\pi\right)$$

$$f'\left(\sqrt{2\pi}\right) = 2\sqrt{2\pi}\cos(2\pi+\pi)$$

$$f'\left(\sqrt{2\pi}\right) = -2\sqrt{2\pi}$$

6. If f is the function given by $f(x)=3x^2-x^3$, then **AROC** of f on [1,5] is

Calculations:

$$f(5)=3\cdot 25-125=75-125=-50$$

$$f(1)=3\cdot 1^2-1^3=2$$

$\frac{-50-2}{5-1}=\frac{-52}{4}=-13$	

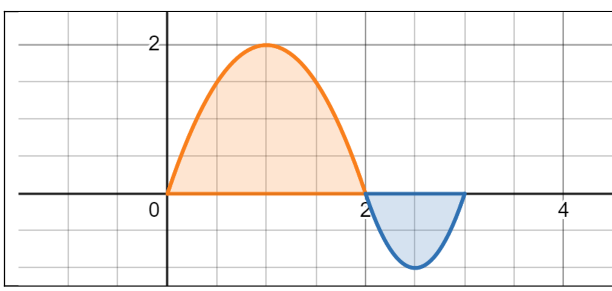
7. If $\int_4^{-10} g(x)\,dx=-3$ and $\int_4^{6} g(x)\,dx=5$, then $\int_{-10}^{6} g(x)\,dx$

Calculations: $\int_{-10}^{4} g(x)\,dx + \int_{4}^{6} g(x)\,dx = \int_{-10}^{6} g(x)\,dx$ $3+5=8$

8. If f is the function given by $f(x)=e^{\frac{x}{3}}$, which of the following is an **equation** of the line **tangent** to the graph of f at point (3 ln 4, 4).

Analysis:	**Calculations:**
Point $(3\ln 4, 4)$ $\quad m=\frac{4}{3}$ $y-4=\frac{4}{3}(x-3\ln 4)$	$f'(x)=\frac{1}{3}e^{\frac{x}{3}}$ *Substituting* $3\ln 4$ *for x* $f'(x)=\frac{1}{3}e^{\frac{3\ln 4}{3}}=\frac{1}{3}e^{\ln 4}$ $f'(x)=\frac{4}{3}$

9. Which of the following expresses the relation between $\int_0^2 f(x)\,dx$, $\int_0^3 f(x)\,dx$, $\int_2^3 f(x)\,dx$

10. $\int_0^2 \left(x^3+1\right)^{\frac{1}{2}} x^2\, dx = ¿$

Calculations:	**Calculations:**
$u=x^3+1 \rightarrow du=3x^2\, dx$ $\frac{du}{3x^2}=dx$ $\frac{1}{3x^2}\int_0^2 \left(x^3+1\right)^{\frac{1}{2}} x^2\, dx$ $\frac{1}{3}\int_0^2 \left(x^3+1\right)^{\frac{1}{2}} dx$	$\frac{1}{3}\left[\frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}}\right]_0^2=\frac{2}{9}\left[\left(x^3+1\right)^{\frac{3}{2}}\right]_0^2$ $\frac{2}{9}\left[(9)^{\frac{3}{2}}-1^{\frac{3}{2}}\right]_0^2=\frac{2}{9}\left[(27)-1\right]=\frac{52}{9}$

11. If $x^2+xy-3y=3$, then at the point (2,1), $\frac{dy}{dx}=¿$

Calculations:	
$2x+x\frac{dy}{dx}+y-3\frac{dy}{dx}=0$ $(x-3)\frac{dy}{dx}=-2x-y$ $\frac{dy}{dx}=\frac{-2x-y}{x-3}$ $Using\ (2,1)$ $\frac{-4-1}{2-3}=5$	

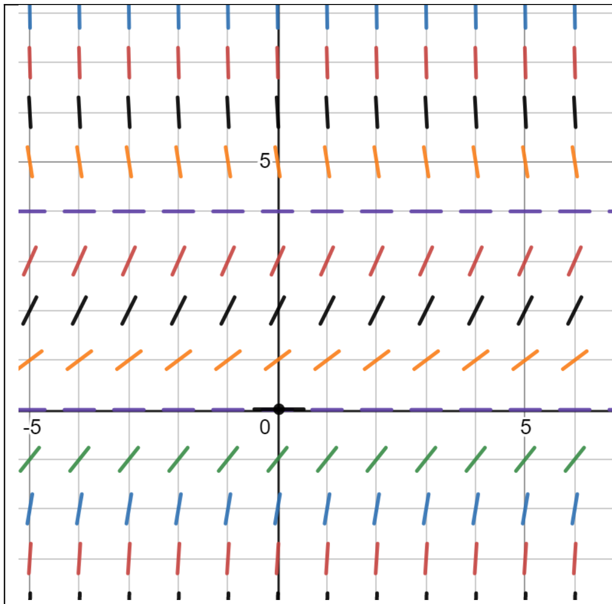
12. The number of gallons of water in a storage tank at time t, in minutes, is modeled by

$w(t)=25-t^2$ for 0≤x≤5. At what **rate**, in gallons per minute, is the amount of water in the tank changing at time t=3?

Verbiage:	**Calculations**:
“at what rate” simply indicates to	$w(t)=25-t^2$

take the derivative. w(t)= number of gallons of water, not a rate.	w'(t)=-2t w'(3)=-6

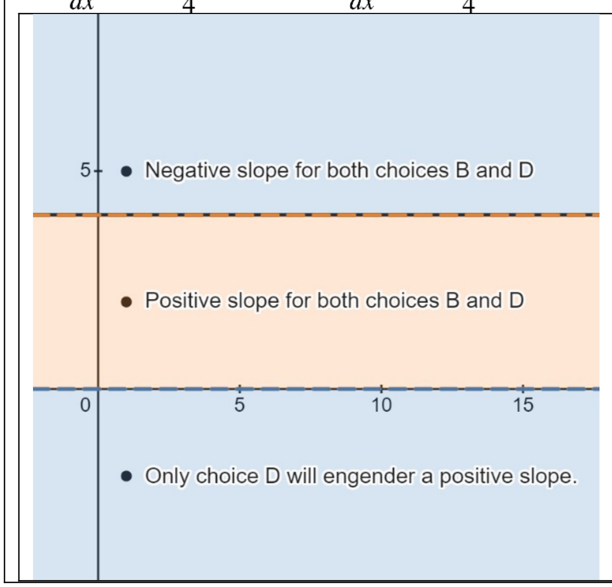
13. Shown is a slope field for which of the following differential equations?



Choices:

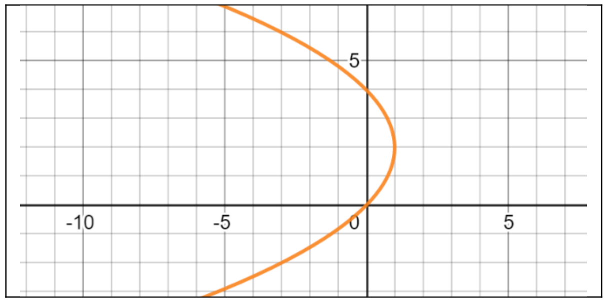
$A.\ \frac{dy}{dx}=\frac{x(4-y)}{4}$ $\quad B.\ \frac{dy}{dx}=\frac{y(4-y)}{4}$

$C.\ \frac{dy}{dx}=\frac{xy(4-y)}{4}$ $\quad D.\ \frac{dy}{dx}=\frac{y^2(4-y)}{4}$



Analysis:

1. The correct answer must include the term $(4-y)$
2. The slope field is independent of x. Thus, choices A&C can be eliminated.



3. Viable choices: B and D.
4. On $y>4$, the slope field should be negative for both choices: B and D.
5. On $0<y<4$, the slope field should be positive for both choices: B and D.
6. On $y<0$, the term $(4-y)$ will be positive. Thus, the value of the slope field depends on y for choice B or y^2 for choice D.
7. Since below the x-axis, the slope field is positive, the only viable choice is D.

Testing Choice B

At $y=5\Longrightarrow\frac{y(4-y)}{4}=\frac{5(4-5)}{4}=-$¿

At $y=3\Longrightarrow\frac{y(4-y)}{4}=\frac{3(4-3)}{4}=+$¿

	$y=-2 \Longrightarrow \frac{y(4-y)}{4}=\frac{-2(4+2)}{4}=-¿$

14. The weight of a population of yeast is given by a differentiable function y, where y(t) is measured in grams and t is measured in days. The weight of the yeast population increases according to the equation $\frac{dy}{dt}=ky$, where k is constant. At t=0, the weight of the yeast population is 120 grams and increasing at the **rate** of 24 grams per day. Which of the following is an expression for y(t)

a) $120e^{24t}$ b) 120 $e^{\frac{t}{5}}$

Analysis:

The weight of a population of yeast is given by a differentiable function , where () is measured in grams and is measured in minutes. The weight of the yeast population increases according to the equation $\frac{dy}{dt}$= , where is a constant. At time = $\mathbf{0}$, the weight of the yeast population is 144 grams and is increasing at the rate of 24 grams per minute. Write an expression for ( ).

24=k(144)

k=$\frac{1}{6}$

$\frac{dy}{dt}$=ky

y=Ce^{kt}

144=C$e^{\frac{1}{6}(0)}$

C=144

y=144$e^{\frac{1}{6}(t)}$

Analysis:	**Calculations**
1. Find k using $\frac{dy}{dt}$ 2. $\frac{dy}{dt}$ =24 since it “is increasing at the rate of 24 grams per day” and y= 120	Using t=0 y=120 $\frac{dy}{dt}$ =24

3. Solving for k, k= $\frac{1}{5}$.
4. It is stated that “at time t=0, the weight of the yeast population is 120”. This implies that y=120
5. Furthermore, it is stated that “(the weight) is increasing at a rate of 24 grams per day”.

   This indicates that $\frac{dy}{dt}=24$
6. Using these facts and the proportion $\frac{dy}{dt}=ky,k=\frac{1}{5}$

$24=120k$

$k=\frac{24}{120}=\frac{1}{5}$

$y=120e^{kt}$

$y=120e^{\frac{1}{5}t}$

Alternative Calculations

$\frac{dy}{dt}=24\,y(0)=120\,y'(0)=24$

$\frac{dy}{dt}=ky$ substituting

$24=k(120)\rightarrow k=\frac{1}{5}$

$\int \frac{dy}{y}=\int k\,dt$

$\ln|y|=kt+C$

$y=e^{kt+C}$

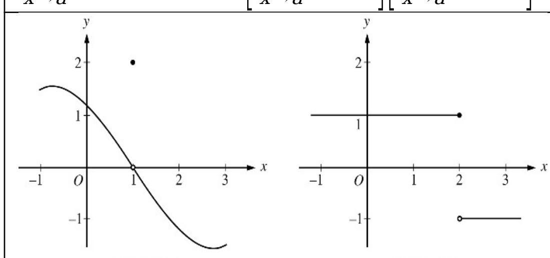
$y=e^{kt}\cdot e^{C}, y=Ce^{\frac{t}{5}}$

$y=120e^{\frac{t}{5}}$

15. The graphs of the functions f and g are show.

Rule:

$\lim_{x\to a}[f(x)g(x)]=\left[\lim_{x\to a}f(x)\right]\left[\lim_{x\to a}g(x)\right]$



Graph of f Graph of g

Analysis:

The key is to find the limit when approaching from the right and from the left.

a) $\lim_{x\to 1}f(x)=0$. This is a true statement.

Analysis:

$\lim_{x\to 1^{+}}$ ¿ $[f(x)]=0$ ¿

and $\lim_{x\to 1^{-}}$ ¿ $[f(x)]=0$ ¿

$\lim_{x\to 1^{+}}$ ¿ $[g(2)]=-1$ ¿

and $\lim_{x\to 1^{-}}$ ¿ $[g(2)]=1$ ¿

Thus,

$\lim_{x\to 1^{+}}$ ¿ $[f(1)g(2)]=(0)(-1)=0$ ¿

$\lim_{x\to 1^{-}}$ ¿ $[f(1)g(2)]=(0)(1)=0$ ¿

b) $\lim_{x\to 2} g(x)$ does not exist.
This is a true statement due to the jump discontinuity.

c) $\lim_{x\to 1}\left[f(x)\,g(x+1)\right]$

d) $\lim_{x\to 1}\left[f(1)\,g(2)\right]$ does not exist is a false statement.

e) $\lim_{x\to 1}\left[f(x+1)\,g(x)\right]$ exits.

f) Both $f(x+1)$ and $g(x)$ have defined limits.

Therefore, $\lim_{x\to 1^{\square}}\left[f(x)\,g(x+1)\right]=0$

The limit approaching a from the left equals the limit when approaching a from the right. So, the limit exits thus making choice C a wrong statement.

C) Left	**C) Right**
$\lim_{x\to 1-¿ f(1)g(2)=0¿} ¿$ $\Longrightarrow\left[(0)(1)\right]=0$	$\lim_{x\to 1+¿ f(1)g(2)=0¿} ¿$ $\Longrightarrow\left[(0)(-1)\right]=0$
D) Left	**D) Right**
$\lim_{x\to 1-¿ f(2)g(1)=-1¿} ¿$ $\Longrightarrow\left[(-1)(1)\right]=-1$	$\lim_{x\to 1+¿ f(2)g(1)=-1¿} ¿$ $\Longrightarrow\left[(-1)(1)\right]=-1$

$\lim_{x\to 1}\left[f(x)\right]$ $\lim_{x\to 1^{+¿}[f(1)]=0¿} ¿$ $\lim_{x\to 1^{-¿}[f(1)]=0¿} ¿$	$\lim_{x\to 1}\left[g(x+1)\right]$ $\lim_{x\to 1^{+¿}[g(2)]=-1¿} ¿$ $\lim_{x\to 1^{-¿}[g(2)]=1¿} ¿$
$\lim_{x\to 1}\left[f(x+1)\,g(x)\right]$ $\lim_{x\to 1^{+¿} f(2)=-1¿} ¿$ $\lim_{x\to 1^{-¿} f(2)=-1¿} ¿$	$\lim_{x\to 1}\left[f(x+1)\,g(x)\right]$ $\lim_{x\to 1^{+¿} g(1)=1¿} ¿$ $\lim_{x\to 1^{-¿} g(1)=1¿} ¿$

16. Let f be the function defined by $f(x)=-3+6x^2-2x^3$. What is the largest open interval on which the graph of f is both **concave up and increasing**?

(-∞,0)	(0,1)	(1,2)	(2,∞)	
neg	pos	pos	neg	f'(x)
pos	pos	neg	neg	f''(x)

Calculations:

$f(x)=-3+6x^2-2x^3$

$f'(x)=12x-6x^2=6x(2-x)$

$f''(x)=12-12x=12(1-x)$

17. A particle moves along the x-axis so that at time t>0 its position is given by

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$s(t)=12e^{-t}\sin t$. What is the **first-time** t at which the **velocity** of the particle is **zero**?

$s(t)=12e^{-t}\sin(t)$ $v(t)=12e^{-t}\cos t-12e^{-t}\sin t$ $12e^{-t}(\cos t-\sin t)=0$ $\frac{\sin t}{\cos t}=1$ $t=\frac{\pi}{4}$	

18. Let F be the function given by $F(x)=\int_3^x [\tan(5t)sec(5t)-1]\,dt$. What is an expression for $F'(x)$?

Analysis: Simply substitute t with x $F'(x)=\tan(5x)sec(5x)-1$	

19. Let f be the function given by $f(x)=2\cos x+1$. What is the **approximation** for $f(1.5)$ found by using the **line tangent** to the graph of f at $x=\frac{\pi}{2}$

Calculations: $f'(x)=-2\sin x$ $f'\left(\frac{\pi}{2}\right)=-2$ $f\left(\frac{\pi}{2}\right)=2\cos\left(\frac{\pi}{2}\right)+1$ $f\left(\frac{\pi}{2}\right)=1$ $Point\left(\frac{\pi}{2},1\right)\ m=-2$	$y-1=-2\left(x-\frac{\pi}{2}\right)$ $y-1=-2\left(1.5-\frac{\pi}{2}\right)$ $y-1=-3+\pi$ $y=-2+\pi$

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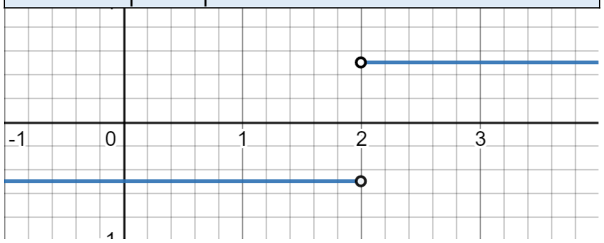
20. Let f be the function given by $f(x)=\frac{x-2}{2|x-2|}$. Which of the following is true?

$f(x)=\frac{x-2}{2|x-2|}$



Choices:

A) $\lim_{x\to 2} f(x) = \frac{1}{2}$.

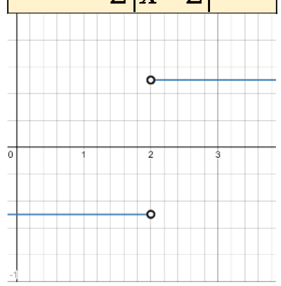
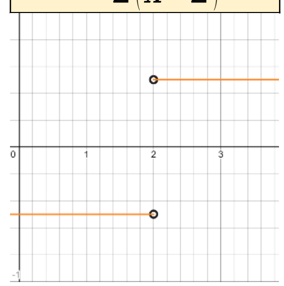
B) f has a removable discontinuity at x=2. False: Jump Discontinuity.

C) f has a Jump Discontinuity

D) f has a discontinuity due to a vertical asymptote at x=2.

Analysis:

1. The graph is in the form of $\frac{(x-a)}{b|x-a|}$. **a** determines where the jump discontinuity will occur.
2. **b** determines how far away from the x-axis is the function located.
3. It does not matter where the absolute value is in the numerator or denominator, the function will look the same.

$f(x)=\frac{x-2}{2\|x-2\|}$	$f(x)=\frac{\|x-2\|}{2(x-2)}$
	

21. If $f(x)=\ln x$, then $\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}$

Definition of Derivative at a Point

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\lim_{x\to 3} ❑ = \frac{\ln x-\ln 3}{x-3}=\frac{\ln 3-\ln 3}{3-3}=\frac{0}{0}$$

Calculations:

$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x} = \frac{1}{3}$$

$\lim_{x\to 3} \square = \frac{\frac{1}{x}}{1} = \frac{1}{3}$	

22. $\frac{dy}{dx} = \frac{2y}{2x+1}$, with the initial condition $y(0)=e$

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Calculations:
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$u = 2x+1 \rightarrow du = 2\,dx$

$dx = \frac{du}{2}$

$\frac{dy}{dx} = \frac{2y}{2x+1}$

$\int \frac{dy}{y} = \frac{1}{2} \int \frac{2}{2x+1}$

$\int \frac{dy}{y} = \int \frac{1}{2x+1}$

$\ln y = \ln \left|2x+1\right|$

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Using $(0, e)$

$\ln e = \ln(1) + c$

$c = 1$

$\ln|y| = \ln(2x+1) + 1$

$e^{\ln|y|} = e^{\ln|2x+1|} + e^{1}$

$y = (2x+1)\,e$

$y = 2ex + e$

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Analysis:

Exponentiate before solving for C.

Alternative Calculations

Point $(0, e)$

$\int_{\square}^{\square} \frac{dy}{2y} = \int_{\square}^{\square} \frac{dx}{2x+1}$

$\frac{1}{2}\ln|2y| = \frac{1}{2}\ln|2x+1| + c$

$\ln|2y| = \ln|2x+1| + c$

$\ln|2y| = \ln|2x+1| + \ln \square C$

$2y = C(2x+1)$ Substitute Point

$2(e) = C(1) \quad \Rightarrow C = 2e$

$2y = 2e(2x+1)$

$y = e(2x+1)$

$y = 2ex + e$

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Alternative Calculations:

$\int_{\square}^{\square} \frac{dy}{2y} = \int_{\square}^{\square} \frac{dx}{2x+1}$

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$u = 2y$

$du = 2\,dy$

$dy = \frac{du}{2}$

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$u = 2x+1$

$du = 2\,dx$

$\frac{du}{2} = dx$

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$\frac{1}{2}\ln|2y| = \frac{1}{2}\ln|2x+1| + c$ (multiply by 2)

$\ln|2y| = \ln|2x+1| + c$

$e^{\ln|2y|} = e^{\ln|2x+1| + C}$

$2y = C|2x+1|$

$y = \frac{1}{2}C|2x+1|$

Evaluate for C, using $(0, e)$

$2y = C|2x+1|$

$2e = C|2 \cdot 0 + 1|$

$2e = C$

Using $y = \frac{1}{2}C|2x+1|$

$y = \frac{1}{2}(2e)(2x+1)$

$y = e(2x+1) \rightarrow y = 2ex + e$

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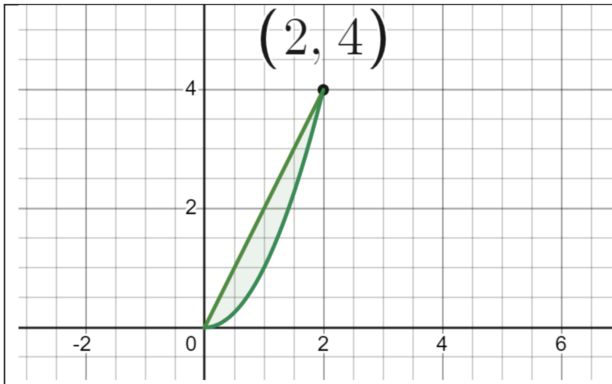
23. The region enclosed by the graphs $y = x^2$ and $y = 2x$ is the base of a solid. For the solid, each cross-section **perpendicular to the y-axis** is a rectangle

whose **height is 3 times its base** in the xy-plane. Which expression gives the **volume** of the solid?



Analysis:

Base: $\sqrt{y} - \frac{1}{2}y$

Height: 3[$\sqrt{y} - \frac{1}{2}y$]dy

x(x-2)=0

Caveat:

(left curve-right curve)

Square the difference and not each term.

Calculations:

$y=2x \rightarrow x=\frac{y}{2}$

$y=x^2 \longrightarrow x=\sqrt{y}$

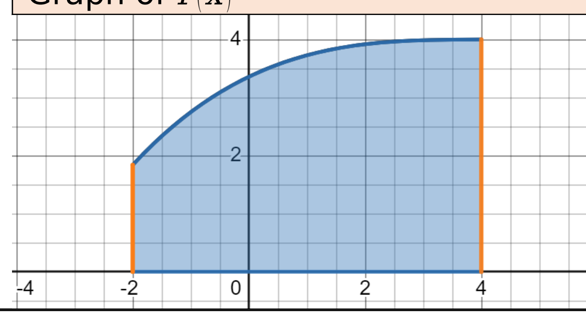
$v=\int_0^4 3\left(\sqrt{y}-\frac{y}{2}\right)^2 dy$

24. If the **average value** of a continuous function f on the interval [-2,4] is 12. What is $\int_{-2}^{4} \frac{f(x)}{8}$dx?

Analysis:

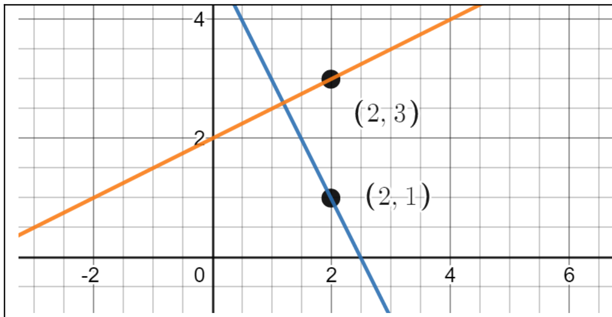
1. The value of the function is (12)(6)=72. The value over the same interval remains 72.

Graph of $f(x)$



25.

The figure shows the graphs of the functions f and g. If h(x)= f(x) g(x), then h’(2)=



Orange: f(x) Blue: g(x)

$f(2)=3$	$g(2)=1$
$f'(2)=\frac{1}{2}$	$g'(2)=-2$

Calculations:

$h(x)=f(x)\ g(x)$

$h'(x)=f(x)\ g'(x)+f'(x)\ g(x)$

$h'(2)=(3)(-2)+\left(\frac{1}{2}\right)(1)$

$h'(2)=\frac{-11}{2}$

26. $\lim_{x\to\infty}\frac{\ln\left(e^{3x}+x\right)}{x}=3$

Analysis:

1. Use l'Hopital's and then use the ratio of leading co-efficients.
2. It is not the definition of the derivative
3. $\ln x$ can be ignored since it grows slower than x.
4. Note that $\ln\left[e^{3x}+x\right]$ can be rewritten as $\ln e^{3x}+\ln x=$ $3x+\ln x$

Alternative Calculations:

$$\lim_{x\to\infty}\frac{\ln\left(e^{3x}+x\right)}{x}$$

$$\lim_{x\to\infty}\frac{\ln e^{3x}+\ln x}{x}=\lim_{x\to\infty}\frac{3x+\ln x}{x}$$

Using l'Hopital's

$$\lim_{x\to\infty}\frac{3+\frac{1}{x}}{1}=3$$

Calculations:

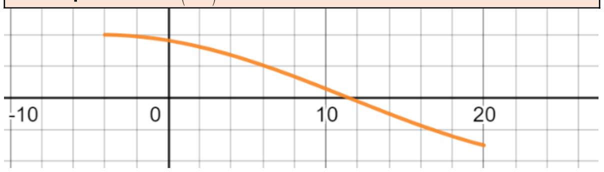
$$\lim_{x\to\infty}\frac{\ln\left(e^{3x}+x\right)}{x}$$

$$\lim_{x\to\infty}\frac{3e^{3x}+1}{e^{3x}+x}$$

$$\lim_{x\to\infty}\frac{9e^{3x}}{3e^{3x}+1}$$

$$\lim_{x\to\infty}\frac{27e^{3x}}{9e^{3x}}=3$$

27. The graph of f' is given. Which of the following statements **MUST** be true?

Graph of $f'(x)$ 	**Analysis:** 1. It is concave down since f'(x) is decreasing. 2. If the derivative is continuous, the function is differentiable. Thus, $f(x)$ is continuous. **Choices:** I. f is continuous on the open interval (a,b). (True) II. f is decreasing on the open interval (a,b). (False). III. The graph of f is concave down on the open interval (a,b). (True).

28. An isosceles right triangle with legs of length s has area $A=\frac{1}{2}S^2$. At the instant when

$s=\sqrt{32}$,the **area** of the triangle is **increasing** at a rate of **12**. At what rate is the length of the **hypotenuse** of the triangle increasing at that instant

Analysis: Given: $s=\sqrt{32}$ $\quad\quad$ $\frac{da}{dt}=12$ -- $h^2=(\sqrt{32})^2+(\sqrt{32})^2=64$ h=8 -- Must find $\frac{ds}{dt}$ first -- Finding $\frac{ds}{dt}$ $A=\frac{1}{2}s^2$	**Calculations:** Finding $\frac{dh}{dt}$ $h^2=s^2+s^2 \Longrightarrow h^2=2s^2$ $2h\frac{dh}{dt}=(2)(2)(s)\frac{ds}{dt}$ $h\frac{dh}{dt}=(2)(s)\frac{ds}{dt}$ $(8)\frac{dh}{dt}=(2)(\sqrt{32})\left(\frac{12}{\sqrt{32}}\right)$ $\frac{dh}{dt}=\frac{24}{8}=3$

$\frac{dA}{dt}=\frac{1}{2}(2s)\frac{ds}{dt}$ $\frac{dA}{dt}=s\frac{ds}{dt}$ $12=\sqrt{32}\frac{ds}{dt}$ $\frac{12}{\sqrt{32}}=\frac{ds}{dt}$	

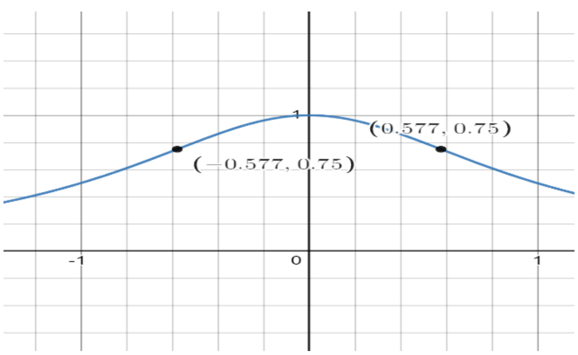
29. The graph of which of the following has exactly **one horizontal** asymptote and **no vertical** asymptote.

Horizontal Asymptote	**Vertical Asymptote**
$\lim_{x\to\infty} f(x)=¿$ That is, f(x) approaches a certain value	$\lim_{x\to¿} ¿ f(x)=¿\infty¿$

A. $f(x)=\frac{1}{x^2+1}$

There is no VA since $x^2\neq-1$

$\lim_{x\to\infty}\frac{1}{(\infty)^2+1}=0$ $\quad$ $\lim_{x\to-\infty}\frac{1}{(-\infty)^2+1}=0$



Calculations:

$f'(x)=\frac{-2x}{(x^2+1)^2}$

$f''(x)=\frac{(x^2+1)^2(-2)+2x(2)(2x)(x^2+1)}{(x^2+1)^4}$ $f''($

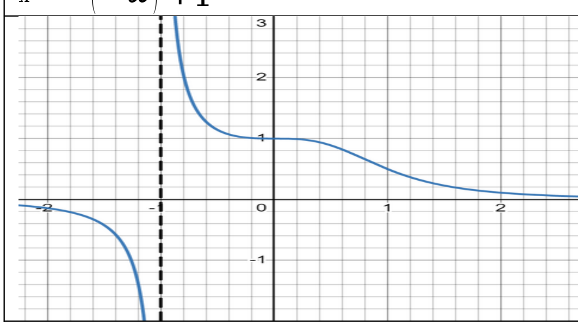
$f''(x)=\frac{(x^2+1)(-2)+8x^2}{(x^2+1)^3}$

B. $f(x)=\frac{1}{x^3+1}$

VA: $x^3=-1\Longrightarrow x=-1$

$\lim_{x\to\infty}\frac{1}{(\infty)^3+1}=0$

$\lim_{x\to-\infty}\frac{1}{(-\infty)^3+1}=0$



$$f''(x)=\frac{6x^2-2}{(x^2+1)^3}=\frac{2(3x^2-1)}{(x^2+1)^3}$$

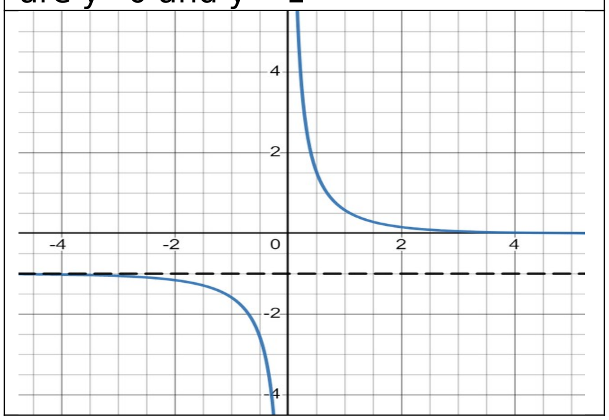
C. $f(x)=\dfrac{1}{e^x-1}$

There is a VA at x=0

$$\lim_{x\to\infty}\frac{1}{e^{\infty}-1}=0$$

$$\lim_{x\to-\infty}\frac{1}{e^{-\infty}-1}=\frac{1}{\frac{1}{e^{\infty}}-1}=-1$$

Thus, the horizontal asymptotes are y=0 and y=-1



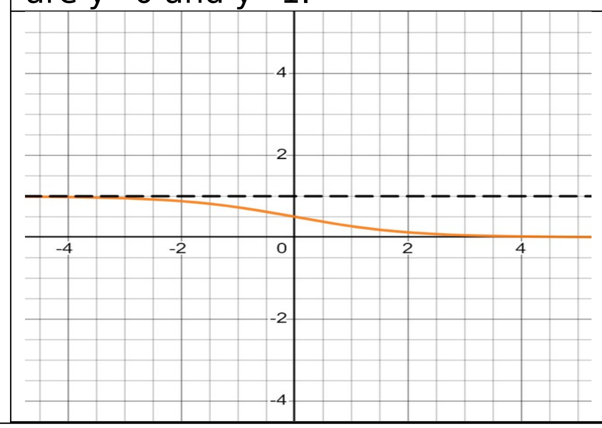
D. $f(x)=\dfrac{1}{e^x+1}$

There is no VA since $e^x\neq 0$

$$\lim_{x\to\infty}\frac{1}{e^{\infty}+1}=0$$

$$\lim_{x\to-\infty}\frac{1}{\frac{1}{\infty}+1}=1$$

Thus, the horizontal asymptotes are y=0 and y=1.



30. For a certain continuous function f, the **RRAM** sum approximation $\int_0^2 f(x)\,dx$ with n subintervals of equal length is $\dfrac{2(n+1)(3n+2)}{n^2}$ for all n. What is the value of $\int_0^2 f(x)\,dx$.

Analysis:	**Calculations:**
The more intervals the closer the Riemann approximation is to the actual area. **Calculations:** $\lim_{n\to\infty}\frac{2(n+1)(3n+2)}{n^2}$	

$\lim_{n\to\infty}\frac{2\left[3n^2+2n+3n+2\right]}{n^2}$ $\lim_{n\to\infty}\frac{2\left[3n^2+5n+2\right]}{n^2}$ $\lim_{n\to\infty}\frac{6n^2+10n+4}{n^2}$**=6**	$\lim_{n\to\infty}\frac{2(n+1)(3n+2)}{n^2}$ $\lim_{n\to\infty}\frac{6n^2+10n+4}{n^2}=6$ $n=1000$ $\frac{2(1001)(3002)}{(1000)^2}=6.010004$ $n=100$ $\frac{2(101)(302)}{(100)^2}=6.1004$ $n=10$ $\frac{2(11)(32)}{(10)^2}=7.02$

76. The graph of a differentiable function f is shown. Which of the following is true.

Analysis: Comparing the slopes of f(x) at x=-2, 0 and 3.	

77. Let **H(x) be an antiderivative** of $\frac{x^3+\sin x}{x^2+2}$. If $H(5)=\pi$, then $H(2)=¿$

Calculations: 1. Need to evaluate $\int_2^5 \frac{x^3+\sin x}{x^2+2}dx$ 2. $\int_2^5 \frac{x^3+\sin x}{x^2+2}dx=H(5)-H(2)$ 3. $H(2)=\pi-(9.00826)=5.867$	**2008 #81** If G(x) is an antiderivative of f(x)and G(2)=-7, then G(4)= $\int_2^4 f(x)=G(4)-G(2)$ $\int_2^4 f(x)+G(2)=G(4)$

78. The continuous function **f is positive** and has a **domain x>0**. If the asymptotes of the graph of f are x=0 and y=2, which statement must be true

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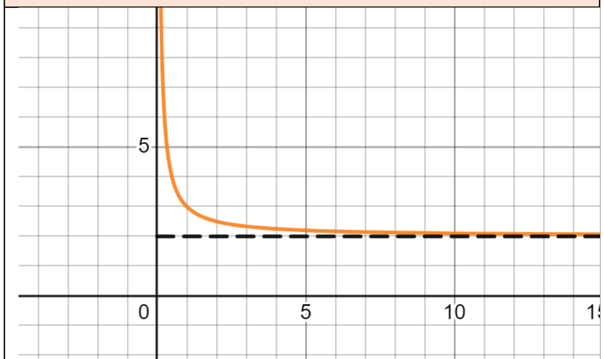
Graph of $y=\frac{1}{x}+2$



Analysis:
$f(x)$ must be above the x-axis.
Domain: $x>0$
Thus, it is restricted to QI

$\lim_{x\to0^+} f(x)=\infty$	$\lim_{x\to\infty} f(x)=2$
A. OK	A. $\lim_{x\to2} f(x)=\infty$
B. $\lim_{x\to0^+} f(x)=2$	B. $\lim_{x\to\infty} f(x)=0$
D. $\lim_{x\to2} f(x)=\infty$	D. OK

a) $\lim_{x\to0^+} f(x)=\infty$ indicates that there is a vertical asymptote at x=0, and $\lim_{x\to2} f(x)=\infty$ indicates that there is a vertical asymptote as x=2. [The latter statement is false]

b) $\lim_{x\to0^+} f(x)=2$ (***For horizontal asymptotes x must approach infinity***), and

$\lim_{x\to\infty} f(x)=0$ indicates that there is a horizontal asymptote at y=0.

c) $\lim_{x\to0^+} f(x)=\infty$ indicates that there is a vertical asymptote at x=0, and $\lim_{x\to\infty} f(x)=2$ indicates that there is a horizontal asymptote at y=2.

d) $\lim_{x\to2} f(x)=\infty$ indicates a vertical asymptote at x=2, and $\lim_{x\to\infty} f(x)=2$ indicates horizontal asymptote at y=2... This is wrong since the VA must be located at x=0 and not x=2.

Analysis:
1. f is positive means that f must lie above the x-axis.
2. The domain x>0
3. Asymptotes at x=0 and y=2
4. If as x→a y→∞, *then* there is a vertical asymptote at x=a
5. If as x→∞ y→a, then there is a

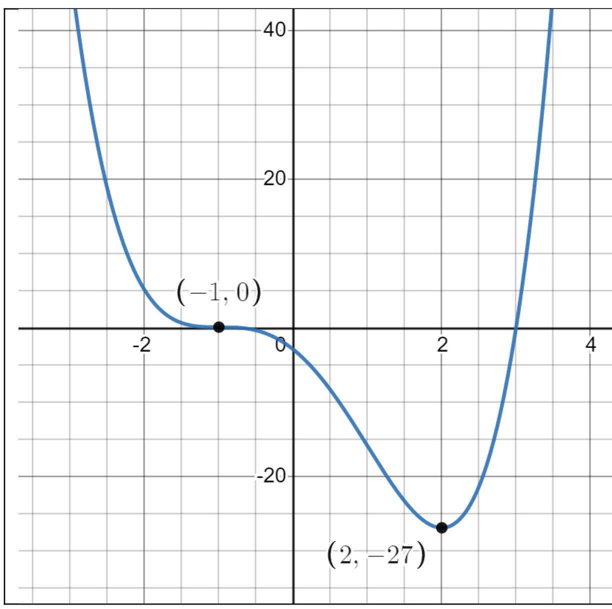
Analysis
- HA at y=2 $\Longrightarrow \lim_{x\to\infty} f(x)=2$. This eliminates choices A&B.
- VA at x=0 $\Longrightarrow \lim_{x\to0} f(x)=\infty$. This eliminates B&D.

horizontal asymptote at x=a 6. x>0 and f>0 will restrict the graph to QI	

79. A file is downloaded to a computer at a **rate** modeled by the differential function f(t), where t is the time in seconds since the start of the download and f(t) is measured in megabits per second. What is the interpretation of f'(5) =2.8. At time t=5, the **rate** at which the file is downloaded to the computer **is increasing at a rate** of 2.8 megabits per second per second.

Analysis: 1. f(t)= rate of download 2. t= time since the download started (in seconds) 3. Megabits per second 4. Must include the verbiage “per second per second “. This excludes choices A and D. 5. Verbiage: “ rate …. is increasing at a rate”	

80. The function f has first derivative given by $f'(x)=x^4-6x^2-8x-3$. On what interval is the graph of f **concave up**?

	**Analysis**:

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81. Given the graph of function f for -2≤x≤2. Which could be the graph of an **antiderivative** of f?

Key: Check for a) where h(x) increases and b) h(x) has HA.
A. Increasing on (-1.5, 1.5)   decreasing: $(-\infty,-1.5)$ and $(1.5,\infty)$
B. Horizontal asymptotes at x= - 1.5. & x=1.5
C. Verbiage: “graph of an antiderivative of f” represents the original function.
D. C is the wrong answer since the graph must increase on (-1.5,1.5).
E. A & B are wrong since the function must be decreasing on (-2,-1.5)

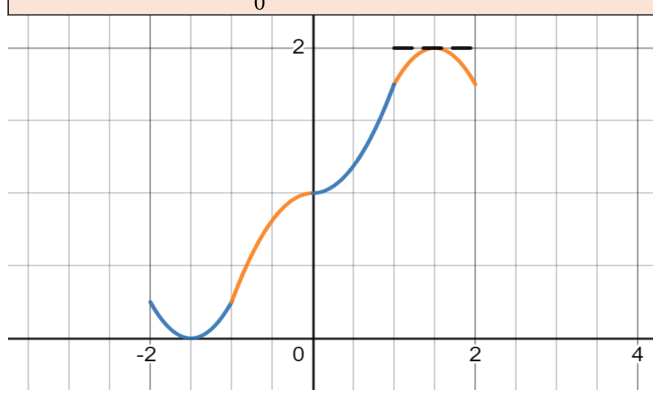
	$(-2,-1.$	$(-1.5,-1$	$(-1,0)$	$(0,1)$	$(1,1.5)$	$(1.5,2)$
$f'(x)$	n	p	P	p	p	n
$f''(x)$	p	p	n	p	n	n

Caveat:

$$h(x)=\int_0^x f(t)\,dt$$

Analysis:
1. Key: check where $f(x)$
• increases
• has HA
2. Must have only two HA. Choice A has three and choices B and C each has 4 HA.
3. All choices have a HA at $x=-1.5$ and $x=1.5$
4. Must decrease first: Thus, eliminating A&B.
5. Between $x=-1.5$and $x=1.5$, must increase. Thus, eliminating C.

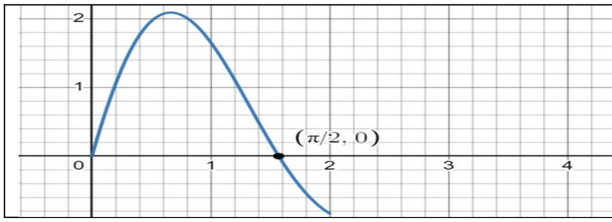
Graph of $h(x)=\int_0^x f(t)\,dt$



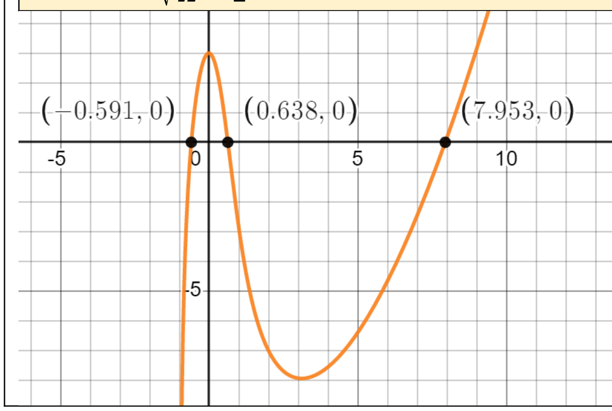
Note:
$h(x)$is an antiderivative of f(x).

82. A particle travels along a straight line with velocity $v(t)=3e^{\frac{-t}{2}}\sin(2t)$ meters per second. What is the **total distance**, in meters, traveled by the particle during [0,2].

$v(t)=3e^{\frac{-t}{2}}\sin(2t)$	**Calculations**:

	$\int_0^2 \lvert v(t) \rvert dt$

83. Let f be a function with derivative $f'(x)=\frac{x^3-8x^2+3}{\sqrt{x^3+1}}$ for (-1,9). At what value of x does f attain a **relative maximum**?

$f'(x)=\frac{x^3-8x^2+3}{\sqrt{x^3+1}}$ 	**Analysis:** Note the open interval

84. The number of bacteria in a container increase at a rate of R(t) bacteria per hour. If there are 1000 bacteria at time t=0, which of the following expressions gives the number of bacteria in the container at t=3.

$1000+\int_0^3 R(t)dt$

85. The function g is continuous on [1,4] with $g(1)=5$ and $g(4)=8$. Of the following conditions, which would guarantee that there is a number C in the **OPEN INTEVAL** (1,4), where g’(c)=1

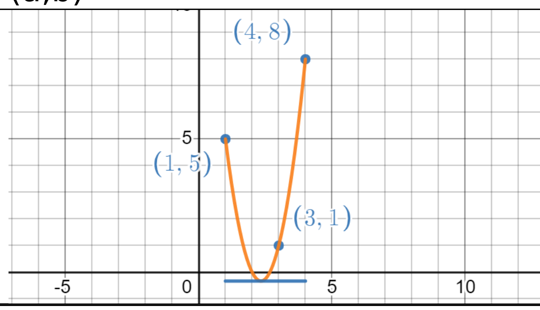
Analysis: Conditions for the **MVT** to apply: a) f must be continuous on [a,b] b) and differentiable on (a,b) Choices: A) $g(x)$Is increasing on the **CLOSED** interval **[1,4]**. It could increase,

but it might not be differentiable at a given point on the interval.

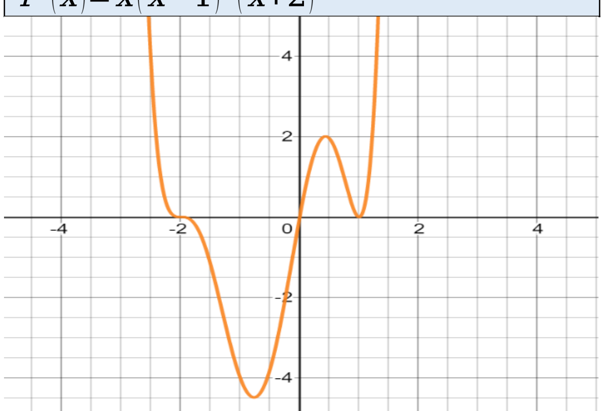
B) $g(x)$ Is differentiable on the **OPEN** interval **(1,4)**. $g(x)$ needs to be differentiable on the open interval for the MVT to apply.

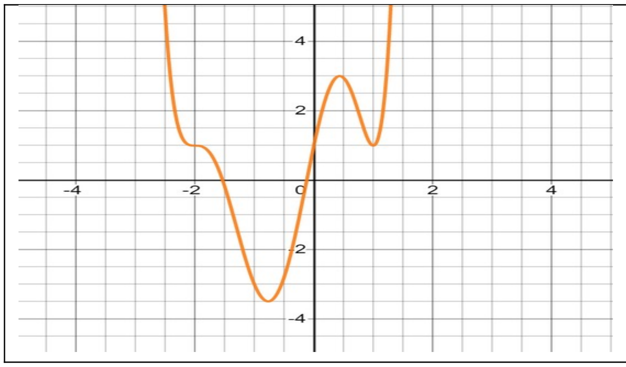
C) $g(x)$ has a maximum value on the **CLOSED** interval **[1,4]**.

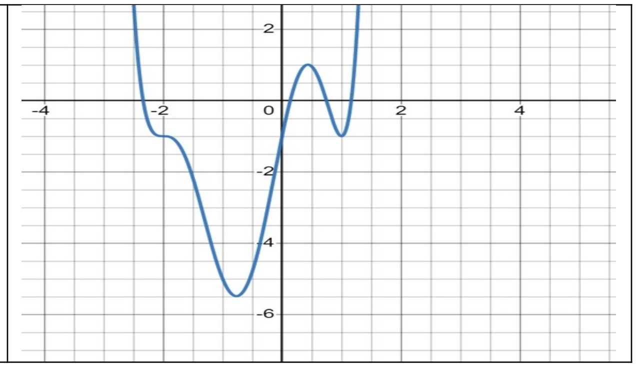
D) The graph of $g(x)$ has at least one horizontal tangent in the **OPEN** interval **(1,4)**. A horizontal tangent does not guarantee g'(c)=1 since a horizontal tangent has a slope of zero.

MVT If f(x) is continuous on a **CLOSED** interval [a,b] and differentiable on the **OPEN** interval (a,b), then there is an x=c on the **OPEN** interval (a,b)	**Calculations:** (1,5),(4,8) $\frac{f(b)-f(a)}{b-a}=\frac{8-5}{4-1}=1$ $m=1$
	

86. The twice differentiable functions f,g,h have the given derivatives. Which function has exactly **two POIs**?

$f''(x)=x(x-1)^2(x+2)^3$ 	Analysis: 1. $f''(x)=x(x-1)^2(x+2)^3$ has exactly two POIs. There is no POI at x=1 since f''(x) does not cross the the x-axis.
$g''(x)=x(x-1)^2(x+2)^3+1$	$h''(x)=x(x-1)^2(x+2)^3-1$



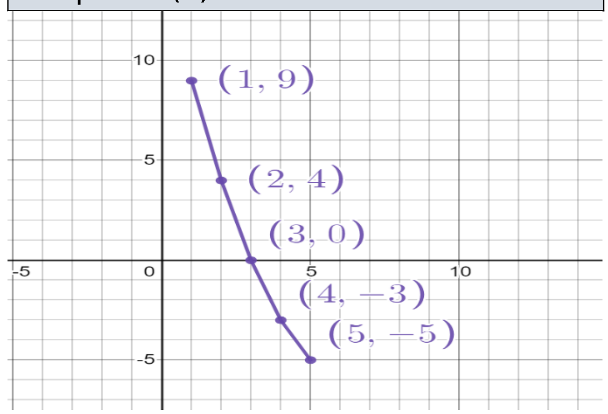


87. If f is twice-differentiable on [1,5], which of the following statements **COULD BE** true based on the table.

x	1	2	3	4	5
f(x)	9	4	0	-3	-5

1. Look at the concavity of the curve! It appears to be CU; thus, f'(x) is increasing.
2. The key is to calculate ,f ’(x)and
   f ''(x). If f ''(x)>0, then f '(x) is increasing.
3. f ’(x) appears to be increasing, I.e. becoming less negative.
4. Note that f(x) appears to be a CU parabola.
5. Thus, f’(x) **COULD BE** negative and increasing for [1,5], that is becoming less negative.
6. Slopes: -5,-4,-3,-2. Thus, when calculated all the slopes are negative which implies that f’(x)<0.
7. The slopes, however, appear to be increasing, i.e., becoming less negative.
8. f''(x)=1 for all values of f''(x) that can be calculated from the given table.
9. Note that (3,0) could be a multiplicity.

Graph of f(x)



Choices: A. f' is negative and decreasing for 1≤x5 B. f' is negative and increasing for 1≤x5 C. f’ is positive and decreasing for 1≤x5 D. f’ is positive and increasing for 1≤x5	
Calculations:	
(1,9) (2,4) $\frac{4-9}{2-1}=-5$	(2,4) (3,0) $\frac{0-4}{3-2}=-4$
(3,0) (4,-3) $\frac{-3-0}{4-3}=-3$	(4,-3) (5,-5) $\frac{-5-(-3)}{5-4}=-2$

88. Let $f(x)=\ln(x^2+1)$ and $g(x)=x^5+x^3$. The line tangent to the graph of f at x=2 is parallel to the line tangent to the graph of g at x=a, where a is a positive constant. What is the value of a?

Analysis:

a) The tangent line to f at x=2 is parallel to the tangent line to g at x=a.
b) f'(2)=g'(a)
c) Find f’(2), which $\frac{4}{5}$
d) Equate g’(x)=$\frac{4}{5}$
e) The slope of the line tangent to f at x=2 is $\frac{4}{5}$
f) Move f(x)=$\frac{4}{5}$x+b to find x=a, where f(x)=$\frac{4}{5}$x+b is tangent to g(x).
g) The slope of the tangent line equals the slope of the f(x) at their POT.

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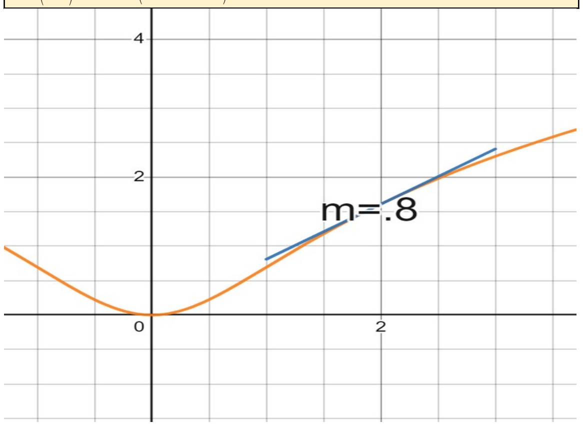
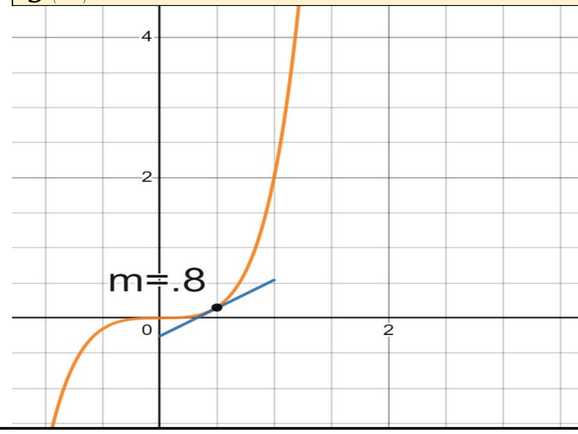
	Calculations	
	$f(x) = \ln\left(x^2+1\right)$ $f'(x) = \dfrac{2x}{x^2+1} \quad \to f(2) = \dfrac{4}{5}$	$g(x) = x^5 + x^3$ $g'(x) = 5x^4 + 3x^2$ $g'(a) = 5a^4 + 3a^2$ $5a^4 + 3a^2 = \dfrac{4}{5}$

$f(x) = \ln\left(x^2+1\right)$



$g(x) = x^5 + x^3$



89. Let f be a **continuous** function for all real numbers. Let g(x)= $\int_1^x f(t)\,dt$. If the **AROC** of g on [2,5] is 6, which of the following statements must be true

Analysis:

1) f(x), which is the derivative of g(x), is continuous. Thus, g(x) is differentiable. This implies that MVT applies.
2) The derivative of g(x), f(x), is continuous.
3) $g(2)$, $g(5)$, $g'(5)$ are unknown since $g(x)$ is unknown.

Choices: Viable Choices

A) The average value of f on the interval 2≤x≤5 is 6. (True):

B) g’(2)=6. This means that the slope at 2 is 6. This cannot be determined.

D) $\int_2^5 g(x)\,dx = 6$ represents the accumulation: The area under the curve. The question

Is asking about **AROC** not the area under the curve.

Calculations:	
$g(x) = \int_1^x f(t)\,dt$ $g'(x) = f(x)$ which is a continuous	$AROC = \dfrac{g(5) - g(1)}{5-2} = 6$ $\left[\dfrac{g(5) - g(2)}{3}\right] = 6$

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function.

Analysis:

$\frac{g'(5)+g'(2)}{2}=6$ is incorrect since the AROC is $\frac{g(5)-g(2)}{5-2}=6$
The original function and its derivative are unknown. Thus, $g(2), g(5), g'(2), g'(5)$ cannot be determined. This eliminates choices B &C.

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$g(5)-g(2)=18$ represents the area under the curve.
Thus, $Average\ Value=\frac{1}{5-2}\int_{2}^{5} f(x)\,dx$

$\frac{1}{3}\int_{\square}^{\square} f(x)\,dx=\frac{1}{3}[f(b)-f(a)]=6$
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Alternative Calculations:
$6=\frac{1}{3}\left[\int_{1}^{5} f(t)\,dt-\int_{1}^{2} f(t)\,dt\right]$
$6=\frac{1}{3}\int_{2}^{5} f(t)\,dt$
Average Value of f on (a,b)
$\frac{1}{b-a}\int_{a}^{b} f(x)\,dx$
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90. For any function which of the following statements **MUST** be true?

Analysis:

I. If f is defined at x=a, then $\lim_{x\to a} f(x)=f(a)$. This is not true since there could be a removable discontinuity on at x=a.

II. If f is continuous at x=a, then $\lim_{x\to a} f(x)=f(a)$. This is the definition of continuity.

III. If f is differentiable at x=a, then $\lim_{x\to a} f(x)=f(a)$. This implies that if a function is differentiable, then it is continuous.

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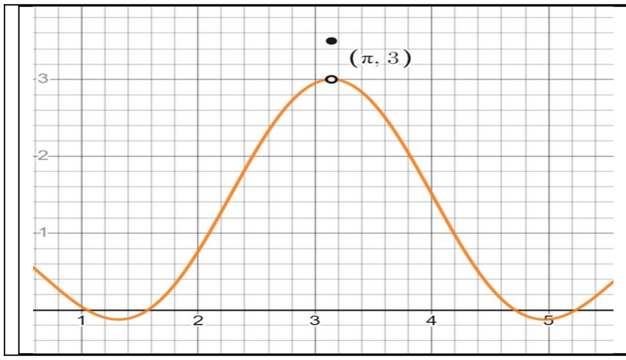	$\lim_{x \to a} f(x) = f(a)$ implies continuity